

Fill in the following identities.

SCORE: ____ / 14 PTS

[a] DIFFERENCE OF ANGLES IDENTITY:

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

[b] SUM OF ANGLES IDENTITY:

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

[c] HALF ANGLE IDENTITY:

$$\sin \frac{1}{2} x = \pm \sqrt{\frac{1 - \cos x}{2}}$$

[d] POWER REDUCING IDENTITY:

$$\cos^2 x = \frac{1}{2} (1 + \cos 2x)$$

[e] NEGATIVE ANGLE IDENTITY:

$$\sec(-x) = \sec x$$

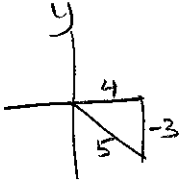
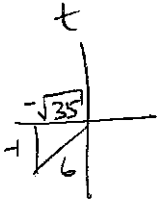
[f] PYTHAGOREAN IDENTITY:

$$\cot^2 x = \csc^2 x - 1$$

[g] DOUBLE ANGLE IDENTITY:

WRITE ALL 3 VERSIONS

$$\cos 2x = \cos^2 x - \sin^2 x \text{ or } 2\cos^2 x - 1 \text{ or } 1 - 2\sin^2 x$$



$$\tan 2t$$

$$= \frac{2 \tan t}{1 - \tan^2 t}$$

$$= \frac{2 \left(\frac{1}{\sqrt{35}} \right)}{1 - \left(\frac{1}{\sqrt{35}} \right)^2}$$

$$= \frac{\frac{2}{\sqrt{35}}}{\frac{34}{35}}$$

$$= \frac{2}{\sqrt{35}} \cdot \frac{35}{34 \cdot 17}$$

$$= \frac{\sqrt{35}}{17}$$

$$\tan \frac{1}{2}t$$

$$= \frac{1 - \cos t}{\sin t}$$

$$= \frac{1 - \left(-\frac{\sqrt{35}}{6}\right)}{-\frac{1}{6}} \cdot \frac{-6}{-6}$$

$$= -6 - \sqrt{35}$$

$$\sin(\underbrace{\arctan(-\frac{3}{4})}_y - t)$$

$$= \sin y \cos t - \cos y \sin t$$

$$= -\frac{3}{5} \cdot -\frac{\sqrt{35}}{6} - \frac{4}{5} \cdot -\frac{1}{6}$$

$$= \frac{3\sqrt{35} + 4}{30}$$

Rewrite $\sin^4 x \cos^2 x$ using only the first powers of cosine (and constants and the 4 basic arithmetic operations). SCORE: _____ / 14 PTS

Simplify your final answer, which must **NOT** be in factored form, and must **NOT** involve any other trigonometric functions.

$$= \left[\frac{1}{2}(1 - \cos 2x) \right]^2 \cdot \frac{1}{2}(1 + \cos 2x)$$

$$= \frac{1}{8}(1 - \cos 2x)(1 - \cos 2x)(1 + \cos 2x)$$

$$= \frac{1}{8}(1 - \cos 2x)(1 - \cos^2 2x)$$

$$= \frac{1}{8}(1 - \cos 2x)\left(1 - \frac{1}{2}(1 + \cos 4x)\right)$$

$$= \frac{1}{8}(1 - \cos 2x)\left(\frac{1}{2} - \frac{1}{2}\cos 4x\right)$$

$$= \frac{1}{16}(1 - \cos 2x)(1 - \cos 4x)$$

$$= \frac{1}{16} - \frac{1}{16}\cos 2x - \frac{1}{16}\cos 4x + \frac{1}{16}\cos 2x \cos 4x$$

Prove the identity $\cot^2 x + \sec^2 x = (\csc x - \tan x)^2 + 2\sec x$.

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$$\hookrightarrow = \csc^2 x - 2\csc x \tan x + \tan^2 x + 2\sec x$$

$$= \cot^2 x + \cancel{1} - 2 \frac{\cancel{1}}{\cancel{\sin x}} \frac{\cancel{\sin x}}{\cancel{\cos x}} + \sec^2 x - \cancel{1} + 2\sec x$$

$$= \cot^2 x + \sec^2 x$$

QED

Solve the equation $9 - 7\cos 5x = 5(2 - \cos 5x)$.

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$$9 - 7\cos 5x = 10 - 5\cos 5x$$

$$-2\cos 5x = 1$$

$$\cos 5x = -\frac{1}{2}$$

$$5x = \frac{2\pi}{3} + 2n\pi \quad \text{or} \quad \frac{4\pi}{3} + 2n\pi, \quad n \in \mathbb{Z}$$

$$x = \frac{2\pi}{15} + \frac{2n\pi}{5} \quad \text{or} \quad \frac{4\pi}{15} + \frac{2n\pi}{5}, \quad n \in \mathbb{Z}$$

Solve the equation $3(4 + \cos 2x) = 5(3 + \cos x)$ algebraically.

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Round your answers to 4 decimal places.

$$12 + 3\cos 2x = 15 + 5\cos x$$

$$12 + 3(2\cos^2 x - 1) = 15 + 5\cos x$$

$$6\cos^2 x + 9 = 15 + 5\cos x$$

$$6\cos^2 x - 5\cos x - 6 = 0$$

$$(3\cos x + 2)(2\cos x - 3) = 0$$

$$\cos x = -\frac{2}{3} \text{ OR } \frac{3}{2}$$

$$X_{\text{REF}} = \cos^{-1} \frac{2}{3} \approx 0.8411$$

$$x \in Q_2 \text{ OR } Q_3$$

$$x = \pi - 0.8411 + 2n\pi, \quad n \in \mathbb{Z}$$

$$\text{OR } \pi + 0.8411 + 2n\pi$$

$$= 2.3005 + 2n\pi$$

$$\text{OR } 3.9827 + 2n\pi$$

$$, \quad n \in \mathbb{Z}$$